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# ELLIPSOMETRY ON A PLANAR S-SHAPE METAMATERIAL

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**Abstract**—In this paper we present the results of ellipsometry on a grid of S-shape particles engraved on an epoxy substrate. We show the value of the ellipticity and the tilt angle of the principal axis of ellipsis of the transmitted wave as a function to the angle between the polarisation of the incident plane wave and the principal axis of the particles. The optical axes of the material are found, and the dependence of the absorption to the angle of polarisation is shown. Using a least squares method with the measurements, we calculate the dichroism and the birefringence of the material.

## 1. INTRODUCTION

Our interest in a bench of measurement for ellipsometry in microwave range, was initially brought by the need of characterization of anisotropic medium like wood, leather or rock. This range of applications was extended to the metamaterials, and we gonna present here the bench of ellipsometry, the metamaterial which was measured and the results of the measurements. The novelty of this work is double. Firstly, even if the idea to use ellipsometry to characterize materials at microwave frequencies is not new [1], its practical realization and measurements, as far as we know, are original in the litteracy. Secondly, the quantification of the anisotropy of a single layer of planar metamaterial in free space by this mean wasn't done before. Such quantification and the identification of optical axis of a planar metamaterial can find application in electronically tunable radomes [2]. Usually, in the

domain of extracting constitutive parameters of metamaterials from measurements, the litteracy presents extraction from the S parameters with the experiment done in the conditions where the material can be considered as isotropic [3, 4]. Finally, the theoretical justifications of the description of a single layer of planar metamaterial by constitutive parameters was done in [5], which justify the measurements of the dichroism and the birefringence of the metamaterial presented here.

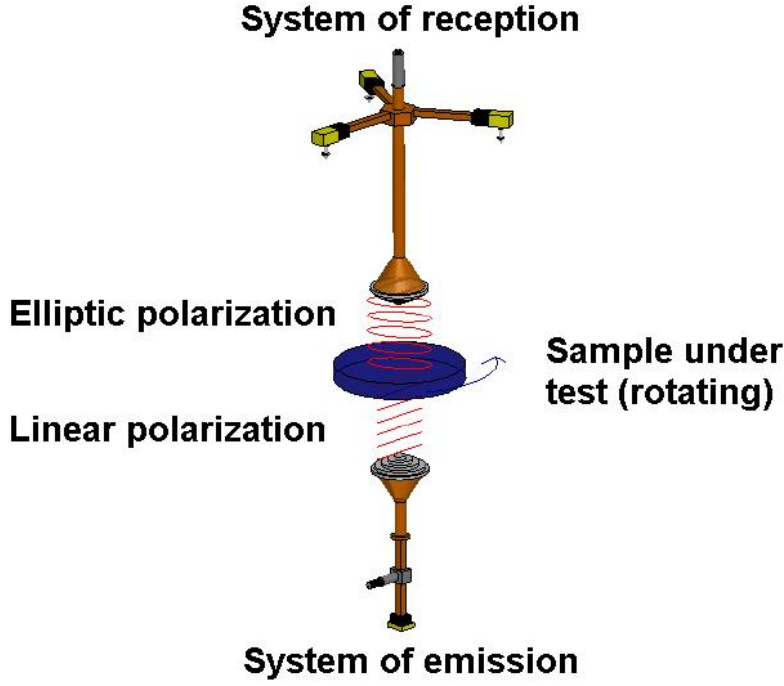
## 2. BENCH OF ELLIPSOMETRY

The principle of our bench of ellipsometry is to make a 3 points measurement on the transmitted wave through the sample in order to determine the dimensions of the ellipsis of polarization. From this measurement, we can determine the value of the dichroism and the birefringence of an anisotropic material [8–13]. The bench is composed of two horns operating at 30 GHz, a Gunn diode and 3 detectors linked to acquisition card. The sample is placed on a rotating part to make the measurements with respect to the angle, rotating clockwise, linked to a computer which drives the motor of the rotating part and get the measurement back from the card (see figure 1). A plexiglas structure surrounds the system, which assure the sample surface to be perpendicular to the axis of propagation of the wave. The two horns are fixed vertically to that structure.

The wave is considered to be plane in the plane of the sample, and the diameter of the beam of 5 cm is smaller than the size of the sample which is 10 cm large. So we consider that we avoid the border effects in the measurement.

## 3. METAMATERIAL SAMPLE UNDER TEST

As a first step, we realized the motif of the array with a commercial software, to make it resonate at 30 GHz by itself and not in its host array, 30 GHz being the work frequency of our free space bench. The loops of the "S particle" have a square shape, as shown in figure 2, so we give the name "square S" to the particle. That type of particle find for example applications in building left-handed material [6], as an alternative to  $\Omega$ -particles [7]. Considering the geometry of the particle, the particle is not bianisotropic because the two loops of opposite sense compensate their effects each others, in a local point of view. It will not either exhibit a magnetic behavior in these conditions, as no magnetic field can excite the loops in the normal incidence. So we consider that the particle will only exhibit an effective permittivity dyadic. The grid of particles was engraved on an epoxy substrate of type FR4 with a

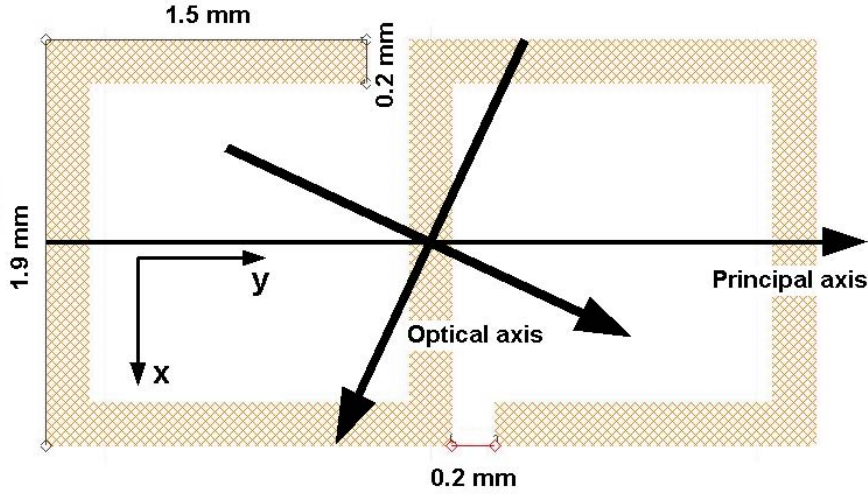


**Figure 1.** General principle of the bench of ellipsometry.

thickness of 1.6 mm and a permittivity of 4.34, which, despite its strong losses at microwave frequencies, has the advantage to allow realizing quick and cheap engraving of circuits, what we needed to realize that study. The dimensions of the sample are those of a square of 10x10 sq cm. The grid has 2 different step values  $d_x = 2.85$  mm in  $x$  direction and  $d_y = 5.40$  mm in  $y$  direction (see Fig. 2 for the definition of the  $x$  and  $y$  axis). These values were chosen to have a compromise between the density of particle in a single cell, which should be small enough to avoid multipole interactions between the particles, and the product  $kd$  (wavenumber multiplied by step of the grid) which should be smaller than 1 to be in the quasi static conditions. These two conditions assure a physical meaning to the calculation of the refractive index. The first condition on the density of particle in a single cell implies that the condition  $d \geq 1.5a$  is satisfied, where  $a$  is the characteristic size of the particles. This condition allows one to neglect the contribution of higher multipoles in static field of a particle at the distance  $d$  from its center. In our case, regarding to the frequency of resonance of

our particle, the respect of the first condition implies that the second condition,  $kd < 1$  assuring the predominance of the static field in the electromagnetic interaction of particles, is not respected a strict way. We got here  $kd_x = 1.79$  and  $kd_y = 3.39$ , nevertheless the measurements of figure 3 show that the the grid is not resonating and so that the quasi-static condition is respected. Firstly the resonating frequency of the sample is that of the single particle and secondly the energy is reflected normally to the plane and not scattered.

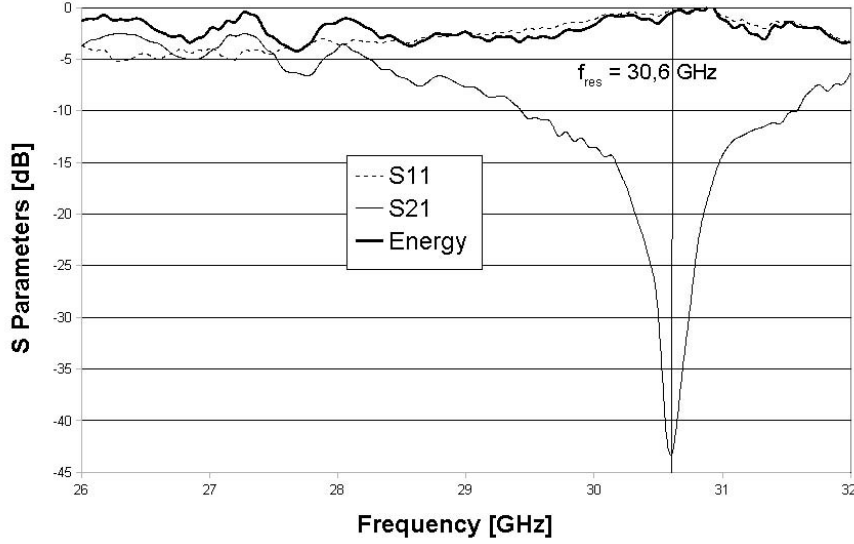
The emitted wave on the sample is linearly polarised, and the beam axis, normal to the sample, gives a plane wave on the sample, as described in the precedent section. The borders effects are minimized, regarding to the size of the beam compared to the sample's one. The diameter of the beam corresponds to 16 cells in the  $x$  direction and to 9 cells in the  $y$  direction.



**Figure 2.** "Square S" particle.

Measurement show a behavior of resonant refractive index with a strong reflection of the wave at the resonance (Fig. 3). This resonance occurs for a certain angle between the polarization of the wave and the particle's main axis, defined as  $\phi$  in this paper, which is shown on figure 4, where stands the transmission parameter with respect to that angle.

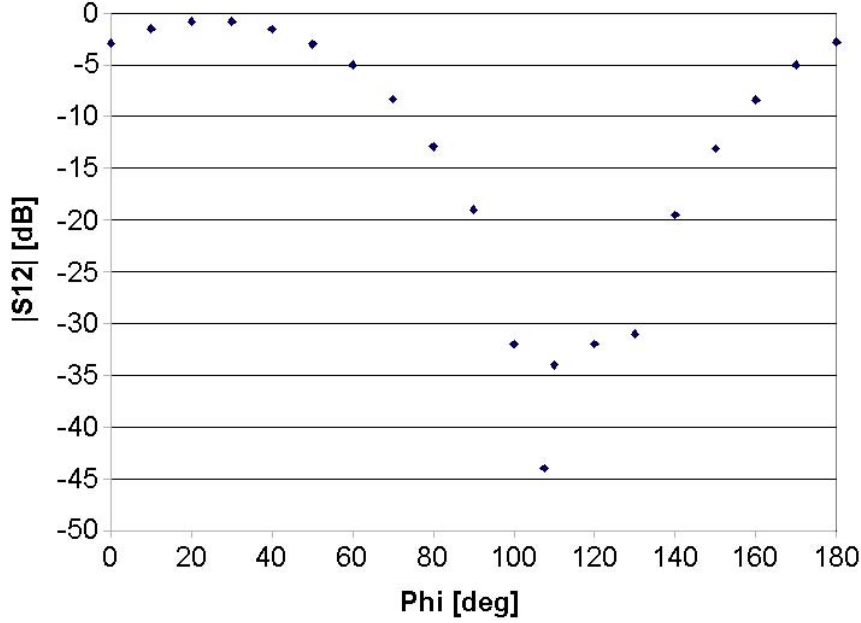
The evaluation of the anisotropy of the material as been conducted, first getting assured that the epoxy substrate itself was not anisotropic in the plane. We can see on figure 5 that the ellipticity of



**Figure 3.** Resonant refractive index effect.

the epoxy is quite constant around 0.2 (ratio  $b/a$  is defined as the small axis  $b$  over the big axis  $a$  of the ellipse). This is a residual value of the ellipticity due to a difference of offset in the tensions delivered by the 3 detectors and it is not significant, as measurement of ellipticity of the air gives the same result. So this result implies that the epoxy substrate is not anisotropic, and is not adding ellipticity to the measurements. The measurements showed that the metamaterial made of square S particles was strongly anisotropic. This anisotropy is evaluated with the measurements of the ellipticity ratio in transmission (figure 6) and with the rotation of the principal axis of the ellipse (figure 7). These measurements allow the evaluation of the variation of  $n'$  and  $n''$  (respectively real and imaginary parts of the refractive index of the material) between the two optical axis.

We can notice regarding figure 6 and 7 that the transmitted wave around the maximum of transmission (for  $25^\circ$  degrees see fig. 4) is polarized linearly, with no tilt on its polarisation. That direction constitute the first optical axis of the material (see Fig. 2). At the particle resonance ( $\phi = 115^\circ$ ), the signal becomes very weak because of the absorption in the resonating metamaterial. We verified that no transmission of energy is done on the cross-polarization, which confirm our hypothesis of absence of bianisotropy. On figure 6, between  $85^\circ$  et  $145^\circ$  degrees, we see that we have a peak tending to an ellipticity



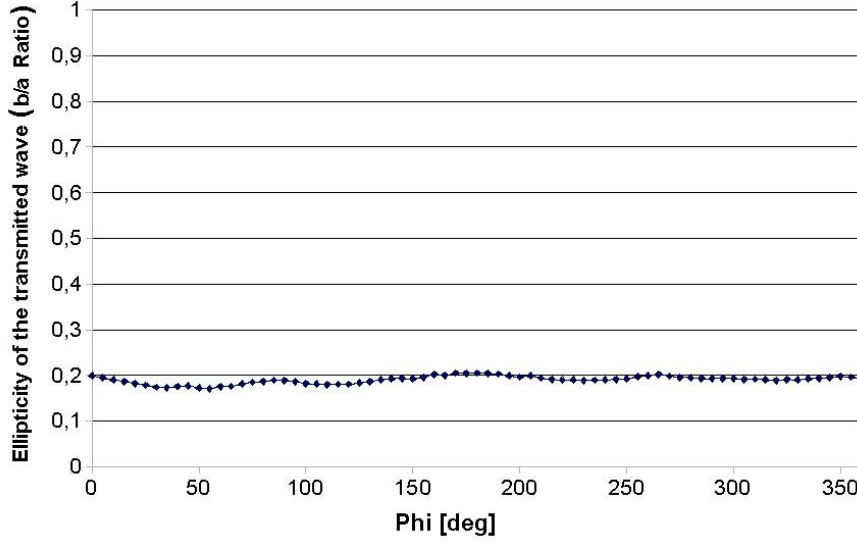
**Figure 4.** S12 with respect to angle of polarisation.

of 1, except at the resonance, where the transmitted wave tends to be linearly polarized, but with a simultaneous effect of the resonance which makes the signal vanish in transmission. The measured signals are then very weak, and the uncertainty in the calculation of the ratio  $b/a$  becomes very big, due to the noise. So the values of the ellipticity for the points of measurements around  $115^\circ$  don't really have a sense. But having a look to figure 4 showing the tilt angle of the principal axis of the ellipsis, we can notice that at the resonance for  $115^\circ$ , this angle is null. So this constitute the second optical axis of our material, distant from  $90^\circ$  to the first one (see Fig. 2). An interesting result stands in the point for  $130^\circ$ , where a quasi circular wave is transmitted.

#### 4. INVERSE PROBLEM

From measurements of the tensions of the 3 detectors of the bench, we can calculate the relative  $\Delta n'$  and  $\Delta n''$ , reciprocally the birefringence and the dichroism of our material.

To determine the birefringence and the dichroism, we use a method of calculation with the help of a software of iteration-convergence [14].

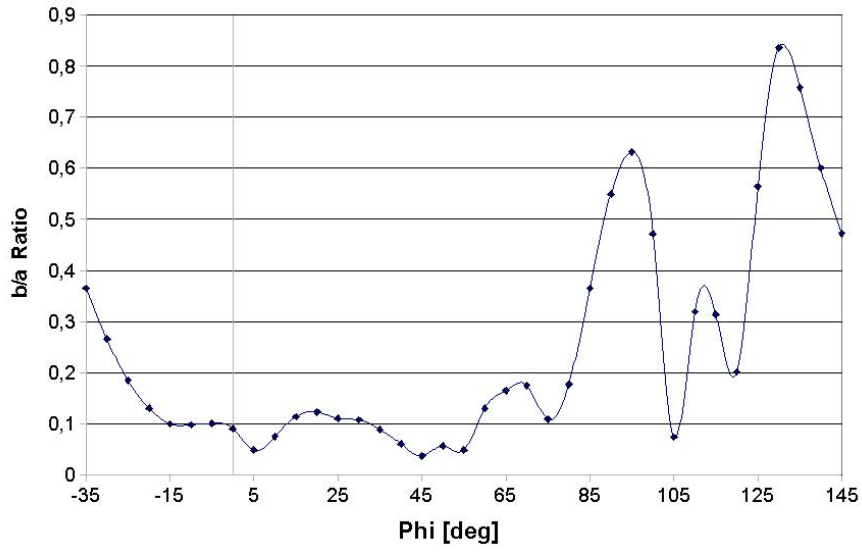


**Figure 5.** Ellipticity of the transmitted wave through the epoxy substrate.

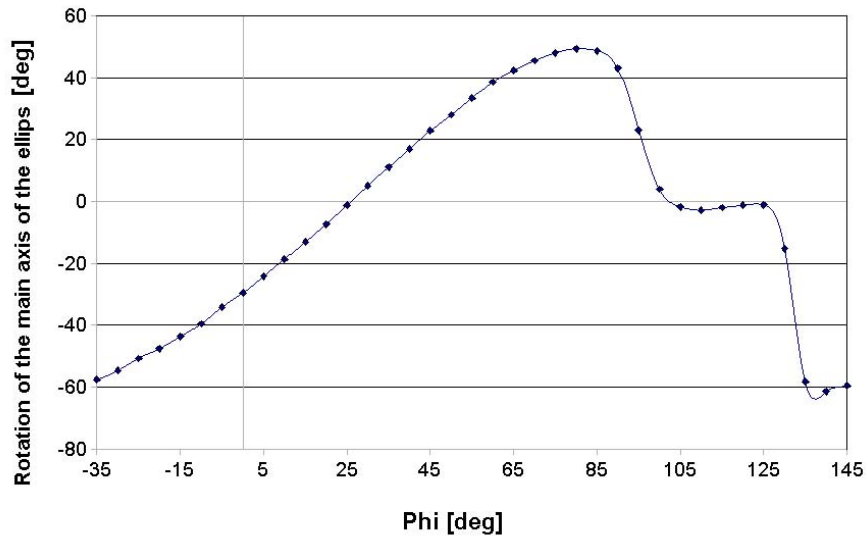
This method is based on the Levenberg-Marquardt's algorithm (least squares method) and permit to find a numerical solution to the problem of minimisation of a function. We then try to minimize the difference between the calculated value of ellipticity and tilt angle of the ellipsis and the measured ones, until the calculated and measured curves finally fit. The uniqueness of the solutions, regarding to the multiple possible periods of the phase of the wave through the sample, is ensured by the small thickness of the layer. Calculation on thicker layers should be of course more prudent, and include considerations on the Riemann Sheets.

The thickness of the metamaterial sample is a parameter needed to calculate the dichroism and the birefringence. For a single layer metamaterial, according to the results developped in [5] the value of the effective thickness is considered as beeing this of the grid cell. Here we take the mean value between  $d_x$  and  $d_y$ , 4 mm. Finally we find the following results, for the dichroism  $\Delta n' = 0.558$  and for the birefringence  $\Delta n'' = -0.210$ , which are strong values as expected with a resonating metamaterial.





**Figure 6.** Ellipticity of the transmitted wave through the metamaterial.



**Figure 7.** Angle of rotation of the principal axis of the ellipsis.

## 5. CONCLUSION

In conclusion, we made a one layer metamaterial composed of so-called square S resonating around 30 GHz, assuming the particle was not bianisotropic nor magnetic. We evaluated the dichroism and the birefringence of that anisotropic material using a method of ellipsometry of the transmitted wave through the sample, coupled to a numerical least square method. The results are showing a very strong anisotropy, as expected with that kind of resonating metamaterial.

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### Figure captions

**Figure 1:** General principle of the bench of ellipsometry.

**Figure 2:** "Square S" particle.

**Figure 3:** Resonant refractive index effect.

**Figure 4:**  $S_{12}$  with respect to angle of polarisation.

**Figure 5:** Ellipticity of the transmitted wave through the epoxy substrate.

**Figure 6:** Ellipticity of the transmitted wave through the metamaterial.

**Figure 7:** Angle of rotation of the principal axis of the ellipsis.